Moduli of general $\mathcal{N} = 1$ heterotic backgrounds

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Motivation

Does string theory describe our universe?

- Heterotic string comes closest to realistic MSSM models

How many realistic string vacua are there?

- Landscape of string vacua depends on choices of theory, geometry, flux, etc.

Can we say anything about *general* compactifications?

- Understanding of moduli spaces

Motivation

Most work done on Calabi–Yau compactifications

- Good for particle physics, YM sector gives Standard Model, compactify on smooth geometries

Relatively easy to find MSSM but generically comes with huge number of moduli

- Moduli are massless fields in low-energy 4d theory
- Need to stabilise these moduli

Motivation

Move away from CY and allow non-zero flux

- Most moduli can be stabilised

Sounds great – what's the catch?

- Internal spaces are not Kähler!
- Little understanding of the moduli spaces

This talk: steps towards a general understanding

Outline

Review of $\mathcal{N} = 1$ compactifications

Strominger system as a holomorphic structure $\bar{D}$

A heterotic superpotential and higher-order deformations

Review of $\mathcal{N} = 1$ compactifications

Heterotic string

Low-energy limit is 10d supergravity coupled to Yang–Mills – "heterotic supergravity"

Want Minkowski compactifications that preserve some supersymmetry

$$M_{10} = \mathbb{R}^{1,3} \times X$$

X is 6d and compact with vector bundle V

- Metric g
- Dilaton ϕ
- Gauge fields A with $G \subseteq \mathrm{E}_8 \times \mathrm{E}_8$
- 3-form flux H

Heterotic supergravity at $\alpha' = 0$

At $\mathcal{O}(1)$ in α', minimal SUSY requires SU(3) holonomy

- X is Calabi–Yau
- No H flux / torsion

Moduli come from deformations of X and V that preserve SUSY

- Kähler, complex structure and bundle moduli
- Generically moduli mix
- How many moduli and how can we stabilise them?

Heterotic supergravity at $\mathcal{O}(\alpha')$

General 4d Minkowski solutions with $\mathcal{N} = 1$ are given by the "Strominger system" [Strominger '86, Hull '86]

- X is complex with an SU(3) structure and a conformally balanced metric
- V and TX are (poly)stable holomorphic bundles
- H satisfies a Bianchi identity (anomaly cancellation)

Difficult to find solutions! [Goldstein–Prokushkin; Fu–Yau; Becker–Sethi; Becker^2 et al.;. . .]

- Torsional geometries not well understood
- Bianchi identity is a headache

Geometry of X

X admits an SU(3) structure

- ω a non-degenerate two-form
- Ω a nowhere-vanishing complex three-form
- $\omega \wedge \Omega = 0$ and $\frac{i}{8}\Omega \wedge \bar{\Omega} = \frac{1}{3!} e^{-4\phi} \omega \wedge \omega \wedge \omega$

X is complex

$$d\Omega = 0$$

The metric on X is conformally balanced

$$d(e^{-2\phi} \omega \wedge \omega) = 0$$

Bundle constraints

Curvature of V defined by

$$F = \mathrm{d}A + A \wedge A$$

F satisfies hermitian YM equations

$$F_{(0,2)} = 0$$
$$F \wedge \omega \wedge \omega = 0$$

V is holomorphic and (poly)stable [Donaldson '85; Uhlenbeck–Yau '86]

Anomaly cancellation

Flux H couples geometry and bundle

$$H = \mathrm{d}B + \frac{\alpha'}{4}(\omega_{\mathrm{CS}}^{A} - \omega_{\mathrm{CS}}^{\nabla})$$

H is torsion of geometry

$$H = \mathrm{i}(\partial - \bar{\partial})\omega$$

H satisfies a Bianchi identity

$$\mathrm{d}H = \frac{\alpha'}{4}(\operatorname{tr} F \wedge F - \operatorname{tr} R \wedge R)$$

where R defined from Hull connection ∇^{-}

SUSY + Bianchi implies equations of motion for this choice of ∇^{-}

Moduli?

What are the moduli of these solutions?

- Many moving parts!
- Can compute in specific cases (worldsheet calculations) [Adams–Lapan '09]
- No systematic understanding until recently

Massless moduli given by infinitesimal problem

Higher-order couplings given by finite problem

Plan

Show that $\mathcal{N} = 1$ solutions are equivalent to existence of a holomorphic structure $\bar{D}$

Moduli live in cohomology of $\bar{D}$

Higher-order deformations controlled by Maurer–Cartan equation

Strominger system as a holomorphic structure $\bar{D}$

Warm-up: complex structure

Complex structure on X equivalent to

$$\mathsf{d} = \partial + \bar{\partial}, \qquad \bar{\partial}^2 = 0$$

Deformations given by $\mu \in \Omega^{(0,1)}(T^{(1,0)}X)$

$$\bar{\partial}' = \bar{\partial} + \mu^a \partial_a$$

Infinitesimal deformations satisfy

$$\bar{\partial}\mu = 0 \quad \Rightarrow \quad \mu \in \mathsf{H}^{(0,1)}_{\bar{\partial}}(T^{(1,0)}X)$$

Finite deformations satisfy Maurer–Cartan equation

$$\bar{\partial}\mu + \frac{1}{2}[\mu, \mu] = 0$$

Warm-up: bundle moduli

X and V have holomorphic structures $\bar\partial$ and $\bar\partial_A$ where

$$\bar\partial^2 = \bar\partial_A^2 = 0, \qquad \bar\partial_A F = 0$$

Note $\bar\partial_A^2 = 0$ equivalent to $F_{(0,2)} = 0$

Define $\mathcal{Q}_1 \simeq \mathrm{End}\, V \oplus T^{(1,0)}X$ (locally)

Holomorphic structure on $\mathcal{Q}_1$ is

$$\bar\partial_1 = \begin{pmatrix} \bar\partial_A & \mathcal{F} \\ 0 & \bar\partial \end{pmatrix} \qquad \text{where } \mathcal{F}(\mu) := \mu^a \wedge F_{a\bar b} \mathrm{d}z^{\bar b}$$

Nilpotency of $\bar\partial_1$ includes Bianchi identity for F

$$\bar\partial_1^2 = 0 \quad \Leftrightarrow \quad \bar\partial^2 = \bar\partial_A^2 = 0, \quad \bar\partial_A F = 0$$

Warm-up: bundle moduli

Want simultaneous deformations of bundle and complex structure [Atiyah '57]

Deform by complex structure by μ and bundle by $(\delta A)_{(0,1)} = \alpha$

$$(\delta F)_{(0,2)} = 0 \quad \Rightarrow \quad \bar{\partial}_A \alpha = \mathcal{F}(\mu)$$

The moduli are $\mathrm{H}^{(0,1)}(\mathrm{End}\, V) \oplus \ker \mathcal{F}$ [Anderson et al. '10]

Expect moduli to be $\mathrm{H}^{(0,1)}_{\bar{\partial}_1}(\mathcal{Q}_1)$

- Can check these agree!
- Gives complex structure and bundle moduli

What did we just do?

1. Define a bundle $\mathcal{Q}_1$ built from X and V
2. Define a differential $\bar{\partial}_1$ so that $\bar{\partial}_1^2 = 0$ iff $\bar{\partial}^2 = \bar{\partial}_A^2 = 0$ and Bianchi for F
3. Moduli given by $\mathrm{H}^{(0,1)}_{\bar{\partial}_1}(\mathcal{Q}_1)$

You can do this for the full $\mathcal{N} = 1$ conditions

- Strominger system equivalent to a holomorphic structure $\bar{D}$ on a bundle $\mathcal{Q}$
- Moduli given by $\mathrm{H}^{(0,1)}_{\bar{D}}(\mathcal{Q})$

[Anderson–Gray–Sharp '14; Garcia-Fernandez '13; Baraglia–Hekmati '13; de la Ossa–Svanes '14]

Strominger system $\equiv$ holomorphic structure

Locally $\mathcal{Q} = T^{(1,0)}X \oplus \mathrm{End}\, V \oplus \mathrm{End}\, TX \oplus \Omega^{(1,0)}(X)$

New ingredient is

$$\mathcal{H}(\mu, \alpha, \kappa) = \mathrm{i}\, \mu^a \wedge (\partial\omega)_{ab\bar{c}} \mathrm{d}z^b \wedge \mathrm{d}\bar{z}^{\bar{c}} - \frac{\alpha'}{4}(\mathrm{tr}\, \alpha \wedge F - \mathrm{tr}\, \kappa \wedge R)$$

where μ, α, κ are $(0,1)$-forms valued in $T^{(1,0)}X$, $\mathrm{End}\, V$ and $\mathrm{End}\, TX$

Construct $\bar{D} = \bar{\partial}_1 + \mathcal{H}$ then

$$\bar{D}^2 = 0 + \text{(poly)stability} + \text{conformally balanced}$$
$$\Updownarrow$$
$$(X, V) \text{ gives } \mathcal{N} = 1 \text{ solution}$$

Massless moduli equivalent to infinitesimal deformations of $\bar{D}$ on $\mathcal{Q}$

Infinitesimal massless spectrum computed by $\mathrm{H}^{(0,1)}_{\bar{D}}(\mathcal{Q})$

$$\mathrm{H}^{(0,1)}_{\bar{D}}(\mathcal{Q}) \cong \ker \mathcal{H} \oplus \left(\mathrm{H}^{(0,1)}_{\bar{\partial}}(T^{*(1,0)}X) \,/\, \mathrm{im}\, \mathcal{H}\right)$$

where $\ker \mathcal{H} \subseteq \mathrm{H}^{(0,1)}_{\bar{\partial}_1}(\mathcal{Q}_1)$

- $\mathrm{im}\, \mathcal{H}$ ensures YM condition for polystable case

A heterotic superpotential and higher-order deformations

Higher-order deformations

At this point we have the infinitesimal deformations

- These deformations can be obstructed at higher orders

In the low-energy theory, the infinitesimal calculation tells you these moduli appear in the action without mass terms

The obstructions at higher orders correspond to Yukawa couplings

We want to find these higher-order contributions

Higher-order deformations

Higher-order deformation problem is difficult

- Complicated and highly dependent on how you parametrise the deformations

Physics guides us

- $\mathcal{N} = 1$ theory $\Rightarrow$ 4d superpotential is holomorphic [McOrist '16]
- Field space is complex with Kähler metric [Candelas et al. '15]
- Superpotential sees only holomorphic deformations
- This tells you how to package the problem

The heterotic superpotential

4d heterotic theory has a GVW-like superpotential [Gukov et al. '99; Becker et al. '03; Cardoso et al. '03, Lukas et al. '05; McOrist '16]

$$W = \int_X (H + \mathrm{i}\, \mathrm{d}\omega) \wedge \Omega$$

Minkowski vacuum $\Leftrightarrow$ $W = \delta W = 0$ on solution

- Recover F-term conditions
- D-term conditions are (poly)stability and conformal balance – not relevant for moduli

Infinitesimal moduli agree with 10d calculation

Suppress TX for now

Parametrising deformations

W is holomorphic so $\bar{\Delta} W = 0$ at generic point in field space

Holomorphic deformations are

$$\begin{aligned}\Delta\Omega &= \imath_\mu \Omega + \tfrac{1}{2}\imath_\mu\imath_\mu\Omega + \tfrac{1}{3!}\imath_\mu\imath_\mu\imath_\mu\Omega,\\ \Delta(B + \mathrm{i}\,\omega) &= x_{(1,1)} + b_{(0,2)}\\ \Delta A &= \alpha_{(0,1)}\end{aligned}$$

Deformed superpotential

Generic holomorphic deformation gives

$$\Delta W = 2\int_X (-\imath_\mu \bar{\partial} x + \tfrac{1}{2}\mathrm{i}\, \imath_\mu \imath_\mu \partial\omega + \ldots - \tfrac{1}{2}\imath_\mu \partial b) \wedge \Omega$$
$$+ \int_X \mathrm{tr}(\alpha \wedge \bar{\partial}_A \alpha - 2\, \imath_\mu F \wedge \alpha + \tfrac{2}{3}\alpha \wedge \alpha \wedge \alpha + \ldots) \wedge \Omega$$

Now want $\Delta W = \delta \Delta W = 0$ for $\mathcal{N} = 1$ Minkowski vacuum

- Still very messy!
- Is there some structure hiding here?

$\bar{D}$ and brackets

Looking for a Maurer–Cartan equation – need a differential and some brackets

Package deformation as

$$y = (x, \alpha, \mu)$$
$$y \in \Omega^{(0,1)}(\mathcal{Q}) \simeq \Omega^{(0,1)}(T^{*(1,0)}X \oplus \mathrm{End}\, V \oplus T^{(1,0)}X)$$

Already have a candidate for the differential: $\bar{D}$

$$(\bar{D}y)_a = \bar{\partial}x_a + \mathrm{i}(\partial\omega)_{ea\bar{c}}\mathrm{d}\bar{z}^{\bar{c}} \wedge \mu^e - \mathrm{tr}(F_{a\bar{b}}\mathrm{d}\bar{z}^{\bar{b}} \wedge \alpha)$$
$$(\bar{D}y)_\alpha = \bar{\partial}_A\alpha + F_{b\bar{c}}\mathrm{d}\bar{z}^{\bar{c}} \wedge \mu^b$$
$$(\bar{D}y)^a = \bar{\partial}\mu^a$$

$\bar{D}$ and brackets

Appearance of $TX \oplus T^*X$ suggests form of bracket

$$
\begin{aligned}
[y, y]_a &= 2\,\mu^b \wedge \partial_b x_a - \mu^b \wedge \partial_a x_b + \dots \\
[y, y]_\alpha &= -2\alpha \wedge \alpha + \dots \\
[y, y]^a &= 2\,\mu^b \wedge \partial_b \mu^a
\end{aligned}
$$

Also have a natural pairing on sections

$$\langle y, y \rangle = 2\,\mu^a \wedge x_a + \operatorname{tr} \alpha \wedge \alpha$$

Deformed superpotential

Deformed superpotential can be written as

$$\Delta W = \int \langle y, \bar{D} y - \tfrac{1}{3}[y, y] - \partial b \rangle \wedge \Omega$$

Generalisation of holomorphic Chern–Simons

$\bar{D}$ and $[\cdot, \cdot]$ satisfy Leibniz identity, and bracket satisfies Jacobi up to ∂-exact terms

Moduli conditions

$\mathcal{N} = 1$ Minkowski vacuum $\Leftrightarrow$ $W = \delta W = 0$ gives

$$\begin{aligned} \bar{D}y - \tfrac{1}{2}[y, y] - \tfrac{1}{2}\partial b = 0, \\ \bar{\partial} b - \tfrac{1}{2}\langle y, \partial b\rangle + \tfrac{1}{3!}\langle y, [y, y]\rangle = 0, \\ \partial \imath_\mu \Omega = 0 \end{aligned}$$

Note that "eqn of motion" for b gives $\partial \imath_\mu \Omega = 0$ – c.f. $\mathrm{d}\Omega = 0$

Solutions give free fields without couplings in the 4d theory

L_3 algebra and field equations

Can be rephrased in terms of L_3 algebra

Combine $Y = (y, b)$ and introduce

$$\begin{aligned} \ell_1(Y) &= (\bar{D}y - \tfrac{1}{2}\partial b, \bar{\partial} b), \\ \ell_2(Y, Y) &= ([y, y], \langle y, \partial b \rangle), \\ \ell_3 &= (0, -\langle y, [y, y] \rangle) \end{aligned}$$

The L_3 field equation reproduces the superpotential conditions

$$\mathcal{F}(Y) = \ell_1(Y) - \tfrac{1}{2}\ell_2(Y) - \tfrac{1}{3!}\ell_3(Y) + \dots$$

Symmetries of moduli encoded as

$$\delta_\Lambda Y = \ell_1(\Lambda) + \ell_2(\Lambda, Y) - \dots$$

Summary

Progress on understanding moduli space of heterotic compactifications

Reviewed infinitesimal moduli in terms of holomorphic structures

Finite deformations of the Strominger system tackled via the 4d superpotential

Simplifies to Chern–Simons like superpotential

Maurer–Cartan equation for L_3 algebra hiding in plain sight

Outlook

Specific examples? Can we compute the cohomologies?

Quantum corrections?

- Superpotential survives α' corrections

Topological theory?

- AKSZ or $\beta\gamma$ worldsheet models [Witten 91], (0,2) models

Non-perturbative effects?

- NS5-branes change Bianchi identity

New invariants?

- Holomorphic CS [Donaldson–Thomas '98]

G_2 backgrounds?

How does heterotic / M-theory duality appear?